# Differential-Jitter Estimation Accuracy Using Centroid-Based Angle-of-Arrival

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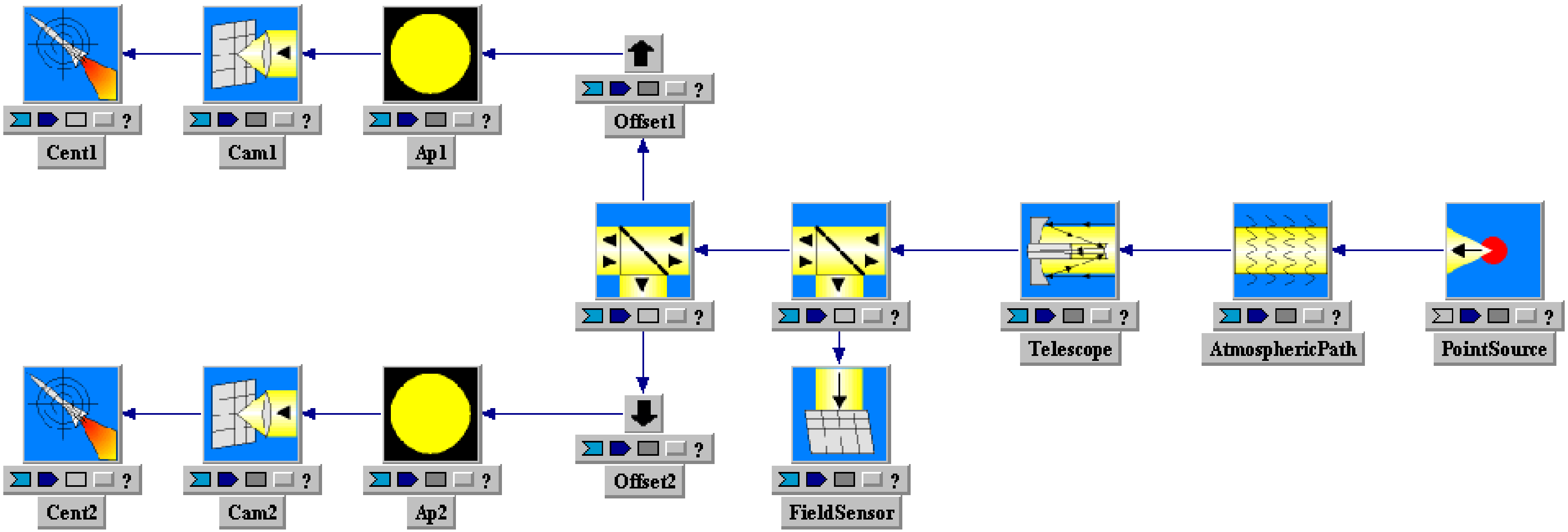
# WaveTrain Model




- **Two cameras offset by an arbitrary separation imaging a common point source**
- **Functionally equivalent to a differential-image-motion monitor (DIMM)**
  - Sometimes referred to as an r0-meter
- **This is the fundamental building block of the LRP**
- **Used 5 cm aperture with closest subaperture spacing from baseline design**
  - Based on SRP, s/D = 1.5875

# Atmospheric & Geometry Modeling Considerations

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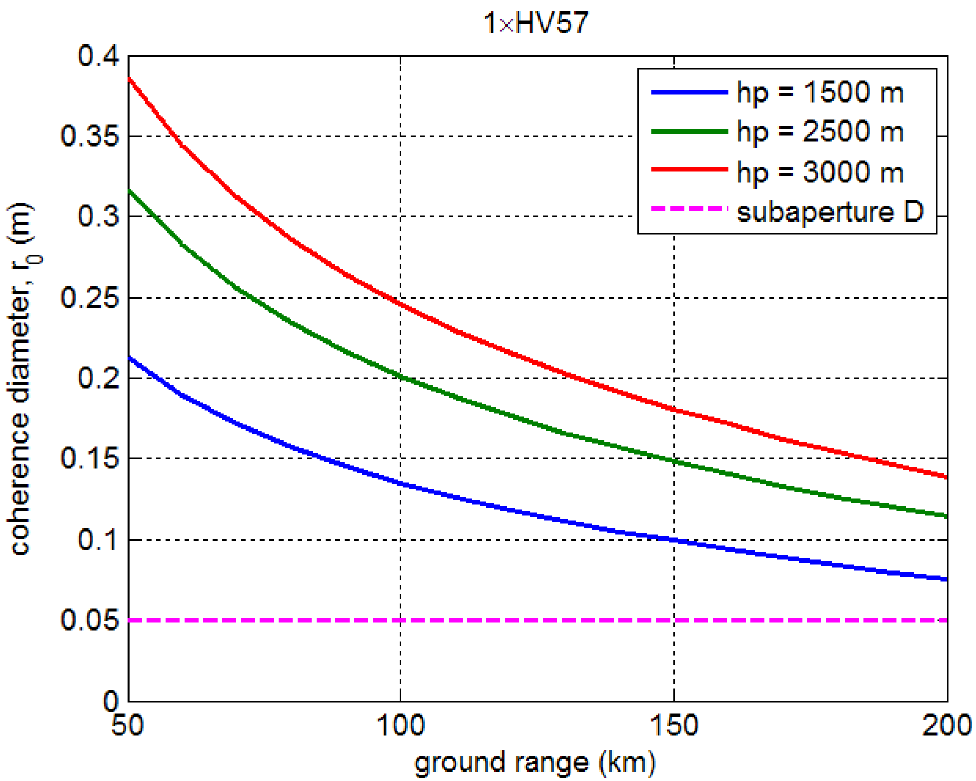


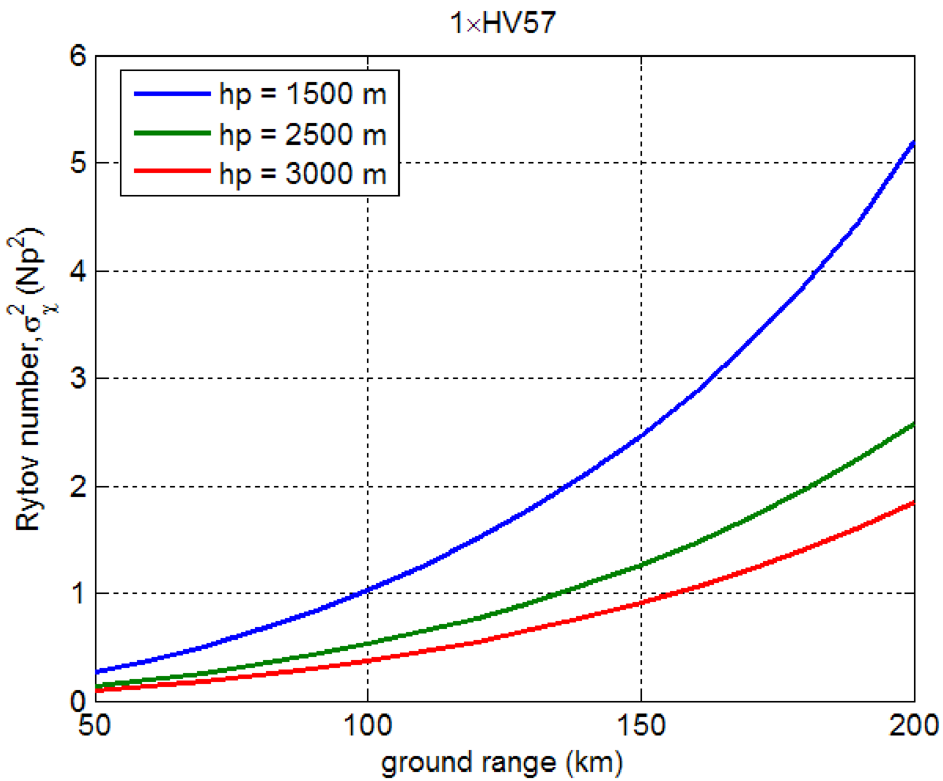


- **Calculations assume λ = 1550 nm**
- **Assumed 1x HV57 turbulence over path with curved earth**
- **r0 larger than baseline subap diameter over altitudes & ground ranges considered**

- **Curved earth causes mid-path turbulence enhancement**
- **Can result in quite high Rytov numbers depending on path altitude and range**
- **Profiler concept based on weak turbulence assumption—practical range limit??**

# Linearity with Turbulence Multiplier

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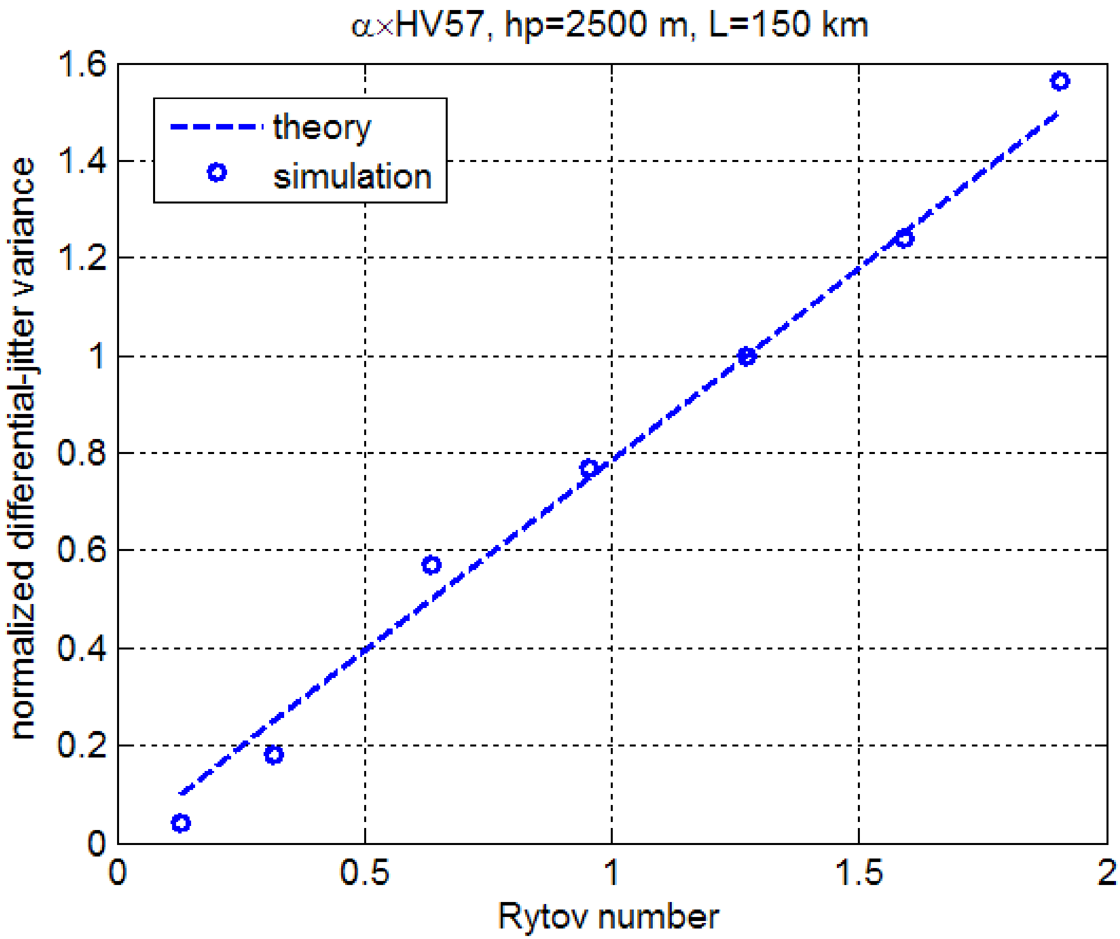
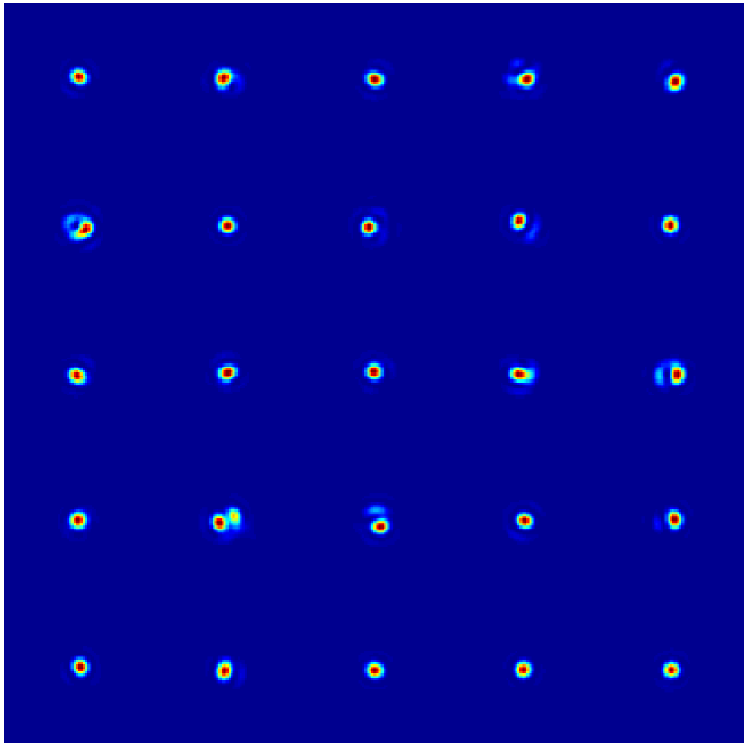


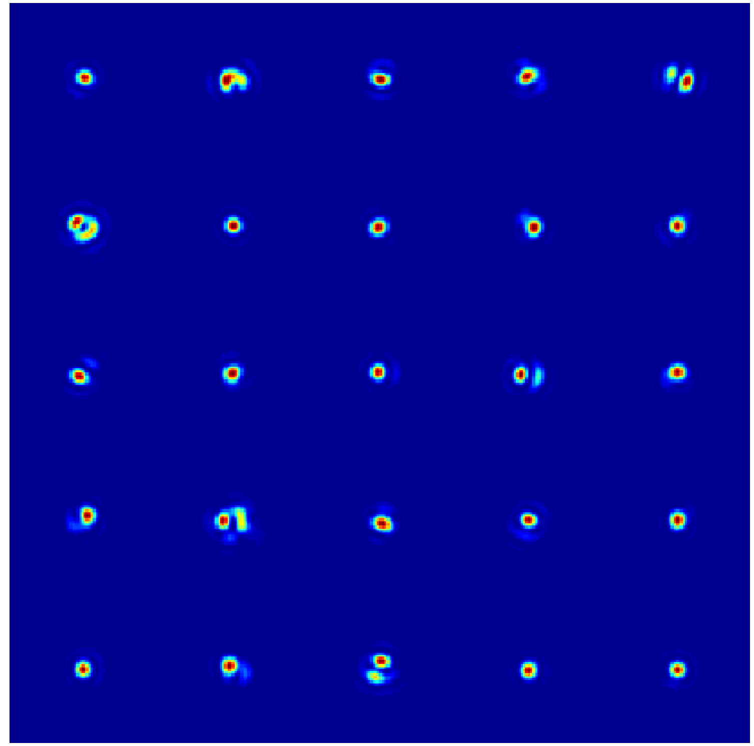

- **Differential jitter variance scales directly with turbulence multiplier**
  - Rytov number also scales directly with turbulence multiplier
- **Using 2500 m altitude and 150 km range we see acceptable linearity of differential-tilt variance**
  - Note: 1x HV57 = Rytov > 1

# Sample Focal Plane Images: HV57 Model

DiffJitterRunHV-2500-150-1p00.trf, 25 realizations



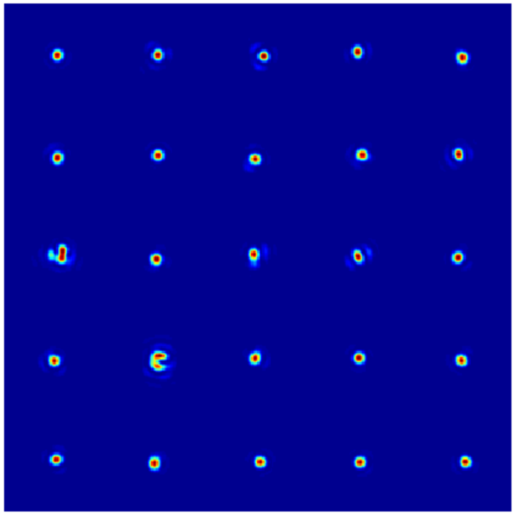
DiffJitterRunHV-2500-150-1p50.trf, 25 realizations



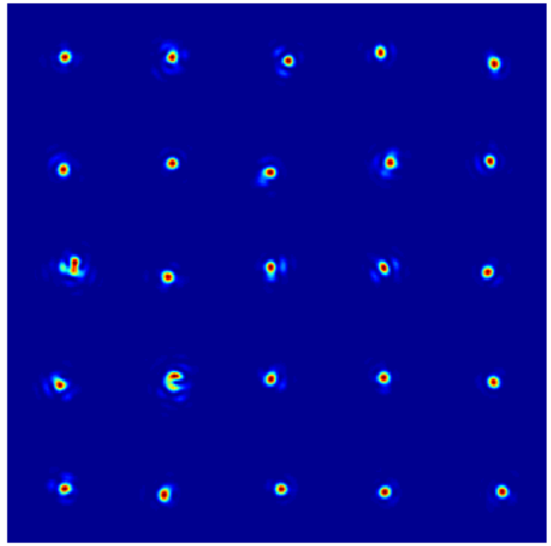
- Focal plane images are well-formed even for Rytov > 1
- Due to the fact that r0 is larger than subaperture

# Sample Focal Plane Images: Slab + HV57 Model

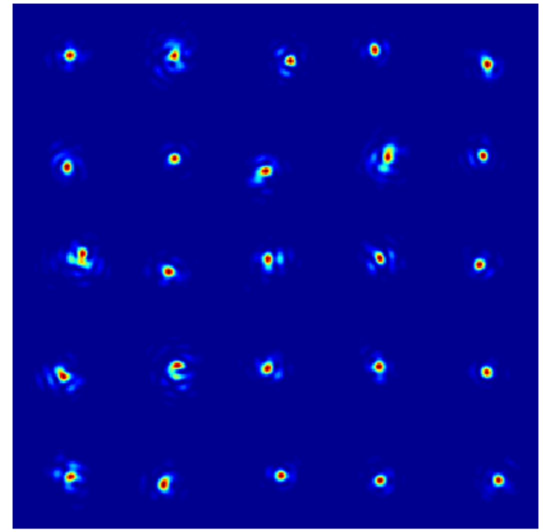
DiffJitterRunSlab10xHV1x.trf, 25 realizations



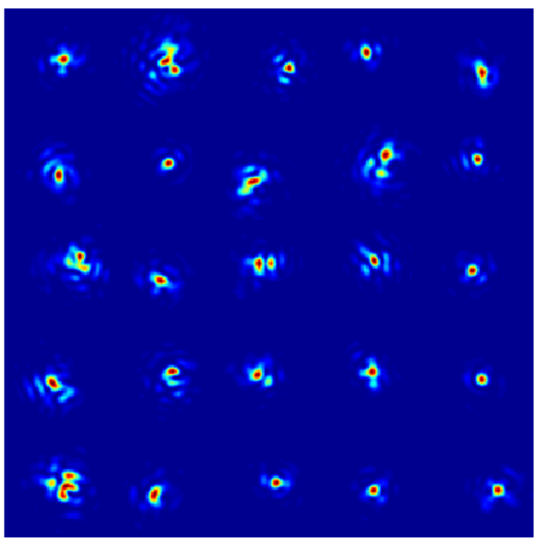
DiffJitterRunSlab50xHV1x.trf, 25 realizations



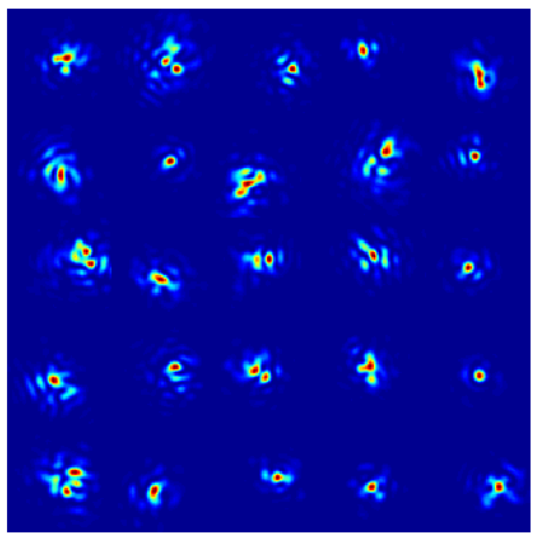
DiffJitterRunSlab100xHV1x.trf, 25 realizations



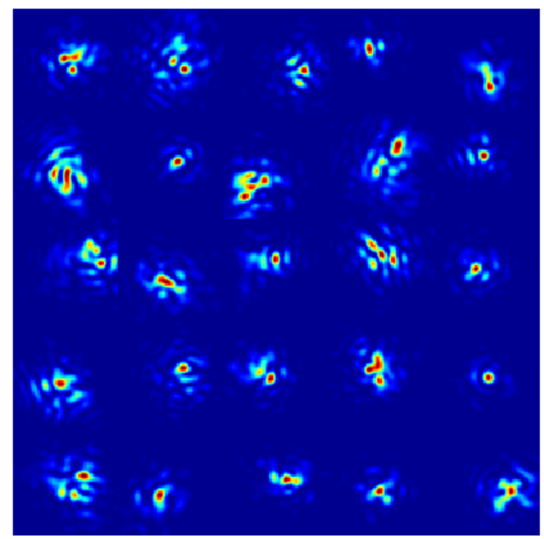
DiffJitterRunSlab200xHV1x.trf, 25 realizations



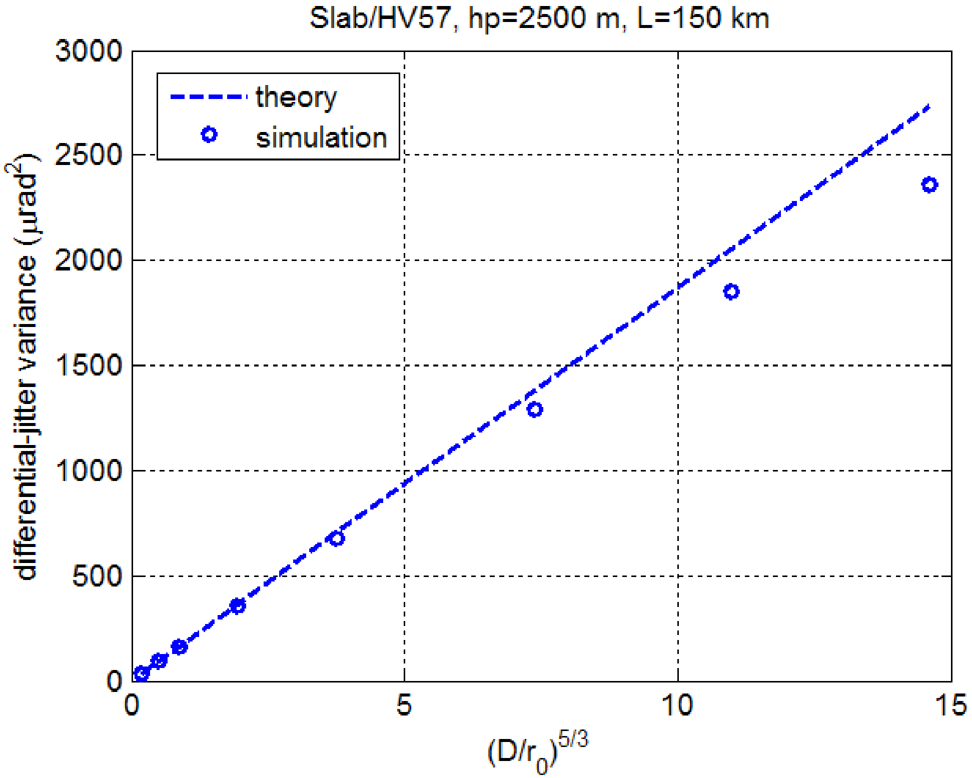
DiffJitterRunSlab300xHV1x.trf, 25 realizations

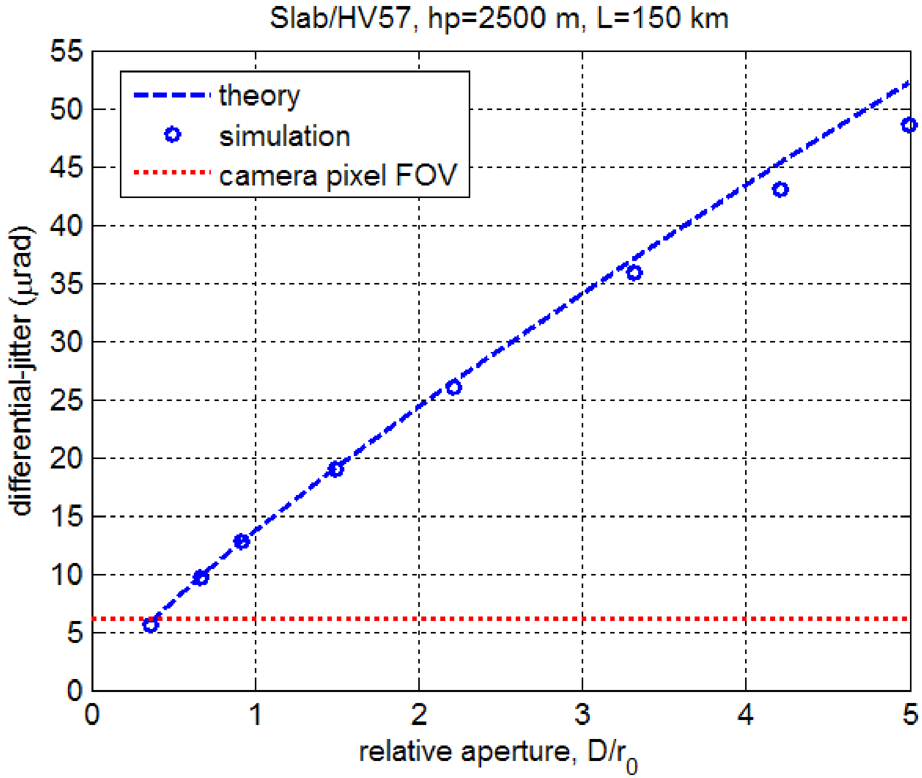


DiffJitterRunSlab400xHV1x.trf, 25 realizations



# Differential Jitter with Slab/HV57




- **Variance compared to theory**
- **Scales linearly with $(D/r0)^{5/3}$**

- **Jitter compared to theory**
- **Plot indicates size of modeled pixel**

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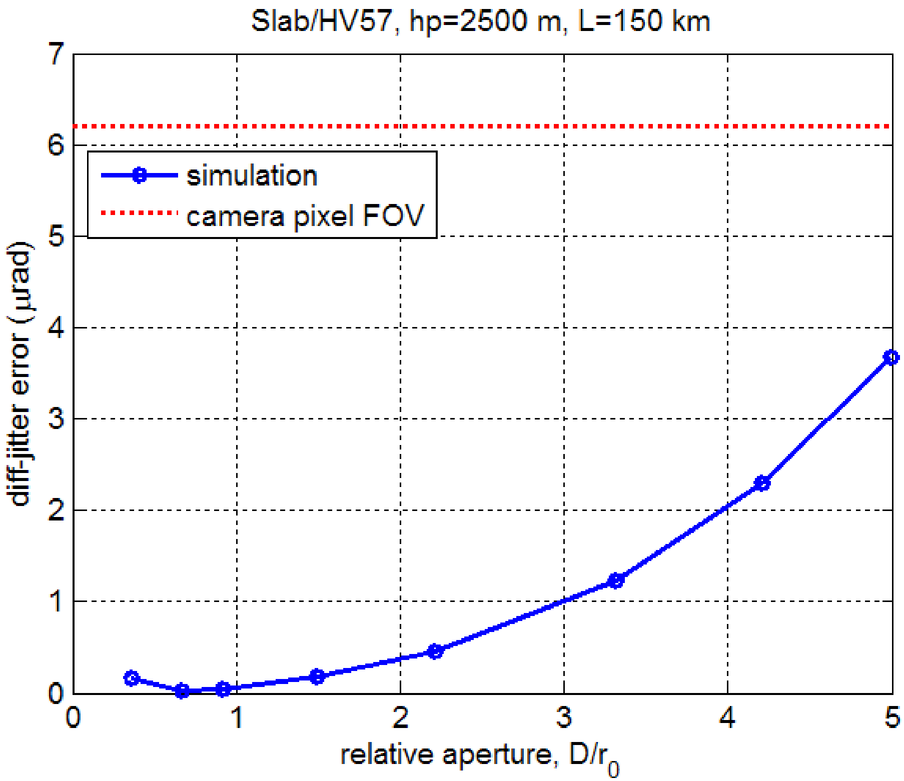
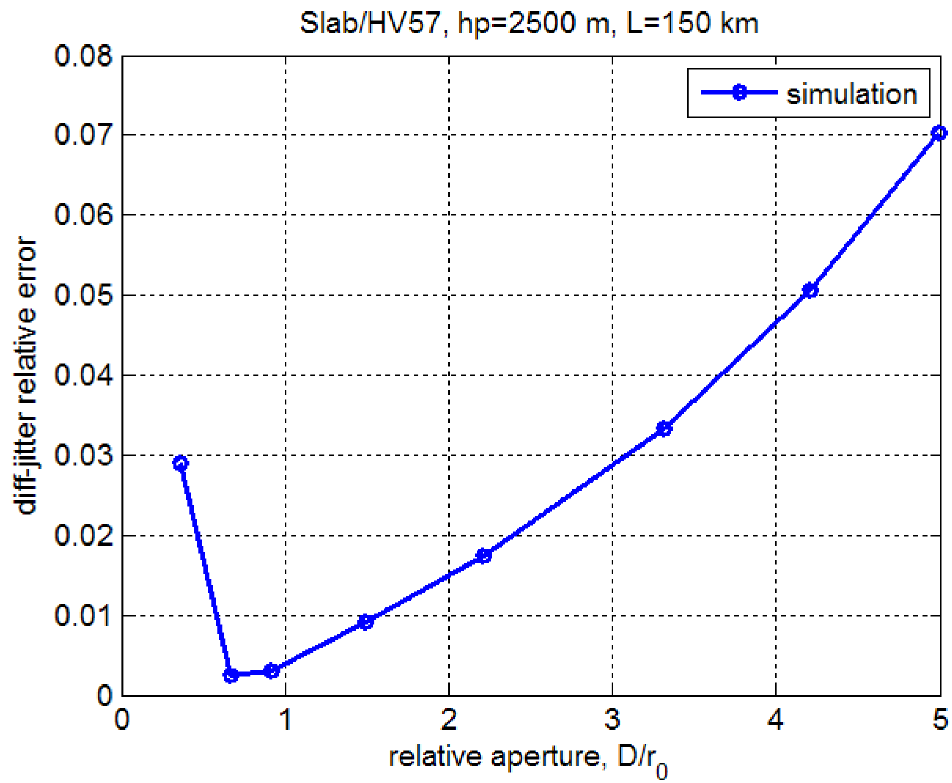
# Differential Jitter Error



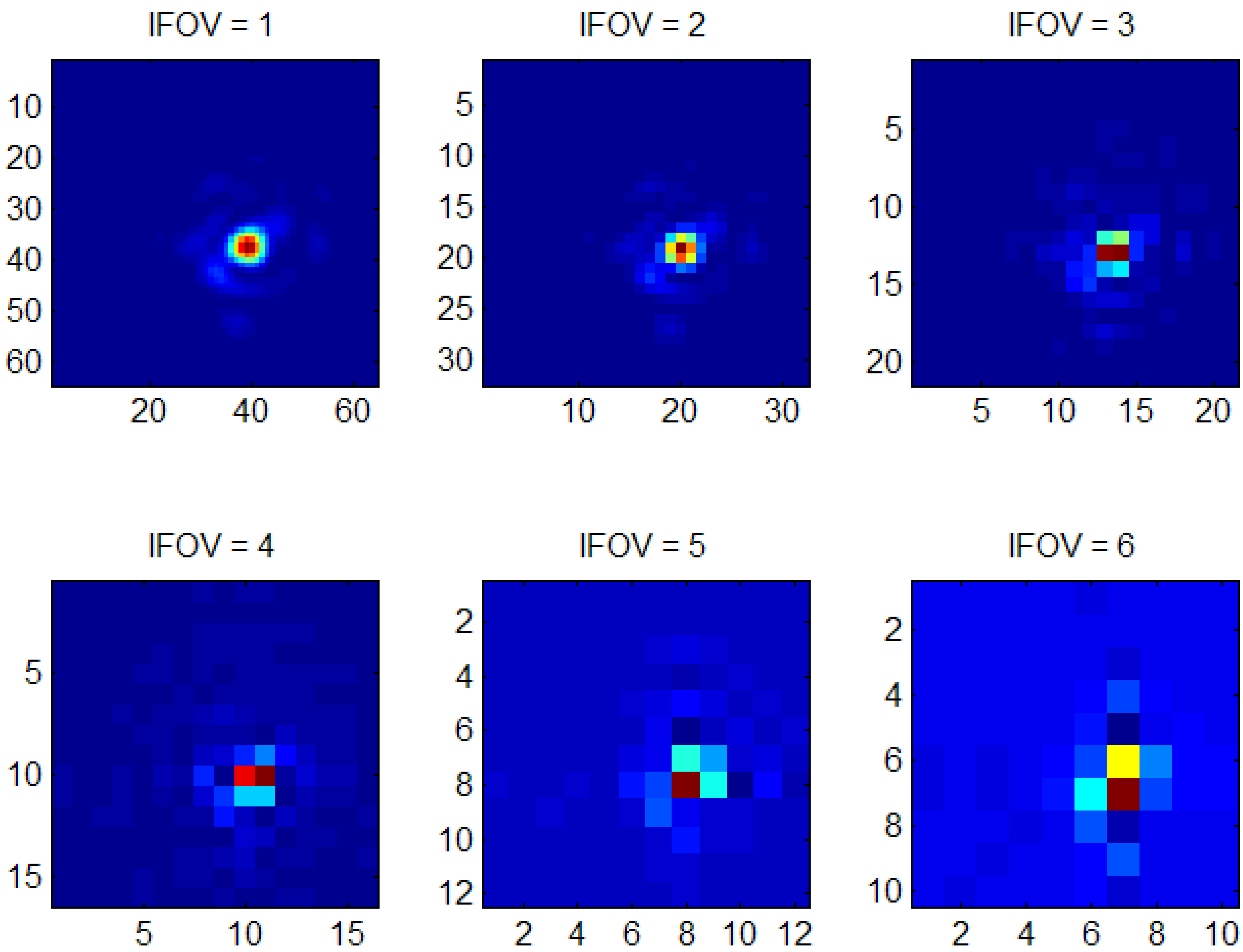




- Error compared to theory is a fraction of the modeled pixel subtense
- Relative error < 10% even for largest values of D/r0
- Centroid error with considerable higher-order breakup is quite tolerable

# Effect of Pixelization on Centroid

- **Centroid accuracy studies were made with “high resolution” imaging system**
  - Camera pixel = 0.2 λ/D → 12 pixels over airy spot
- **Gen-2 profiler design will work with larger pixel IFOV**
  - Allows more light in pixel for long-range operation
- **Centroid will be affected by size of pixel on focal plane**
- **To address this concern**
  - Simulation image data was reprocessed into images with lower resolution
  - Centroid data was recomputed for lower-resolution images
  - Reported centroid was compared with centroid from high-resolution image

# Example: Images with Increased IFOV



# Centroid Accuracy with Slab/HV57

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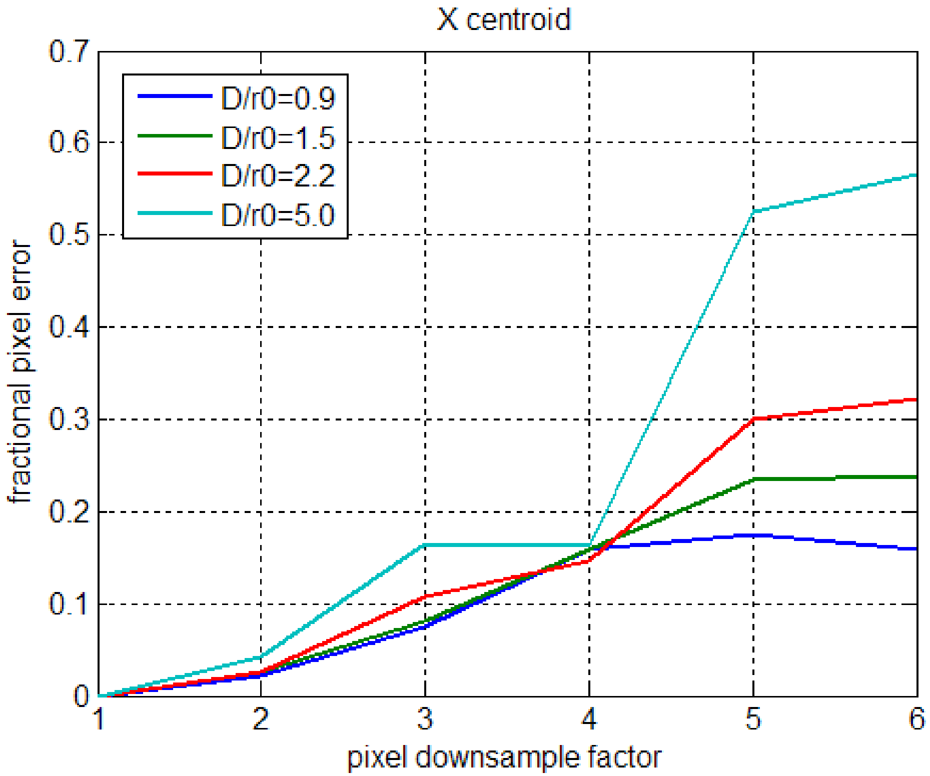


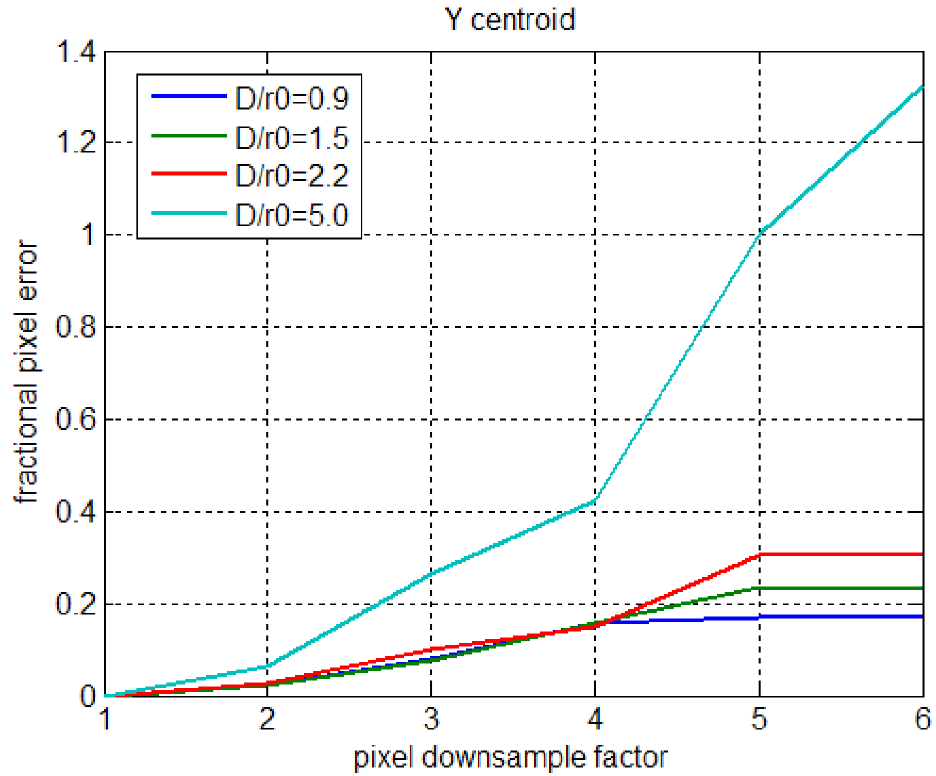


- **X-axis centroids**
- **"Pixel downsample factor" = ratio of IFOV to high-resolution image**
- **high-res = 6.2 μrad = 0.2 λ/D**

- **Y-axis centroids**
- **Error nominally independent of D/r0 up to IFOV = 0.8 λ/D**
- **Pixelization error < 0.1 pixel for IFOV < 0.6 λ/D**
  - <0.6 μrad

# Conclusions

- **Scaling of differential tilt variance with strength multiplier for distributed turbulence shows expected linearity**
- **“Weak turbulence” assumption underlying profiler theory of operation holds up well at high Rytov numbers**
- **Effect of higher-order phase on centroid-tilt accuracy is <10% effect for $D/r_0<5$**
- **No compelling reason to use focal plane processing other than centroid to determine tilt (angle of arrival)**
  - Applying a threshold prior to centroid desirable, as is done with the short-range profiler
- **~20 μrad pixel will result in ~0.5 μrad centroid error due to pixelization**
  - This effect combines with differential-jitter error from theory/simulation comparison
  - <10% error for likely propagation conditions