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Some aspects of wavefront tilt and relative motion in the WaveTrain propagation code

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Background

- **Several WaveTrain users (both internal and external to MZA) have been disturbed by a certain aspect of the wavefront tilt reported by WaveTrain when the receiver is moving relative to the source.**
- **Analysis of the problem from the point of view of different reference frames seems (at first) to have some paradoxical features.**
- **The issue is connected with concepts and methods of special relativity, but we can discuss and resolve the issue without the full relativity analysis machinery.**

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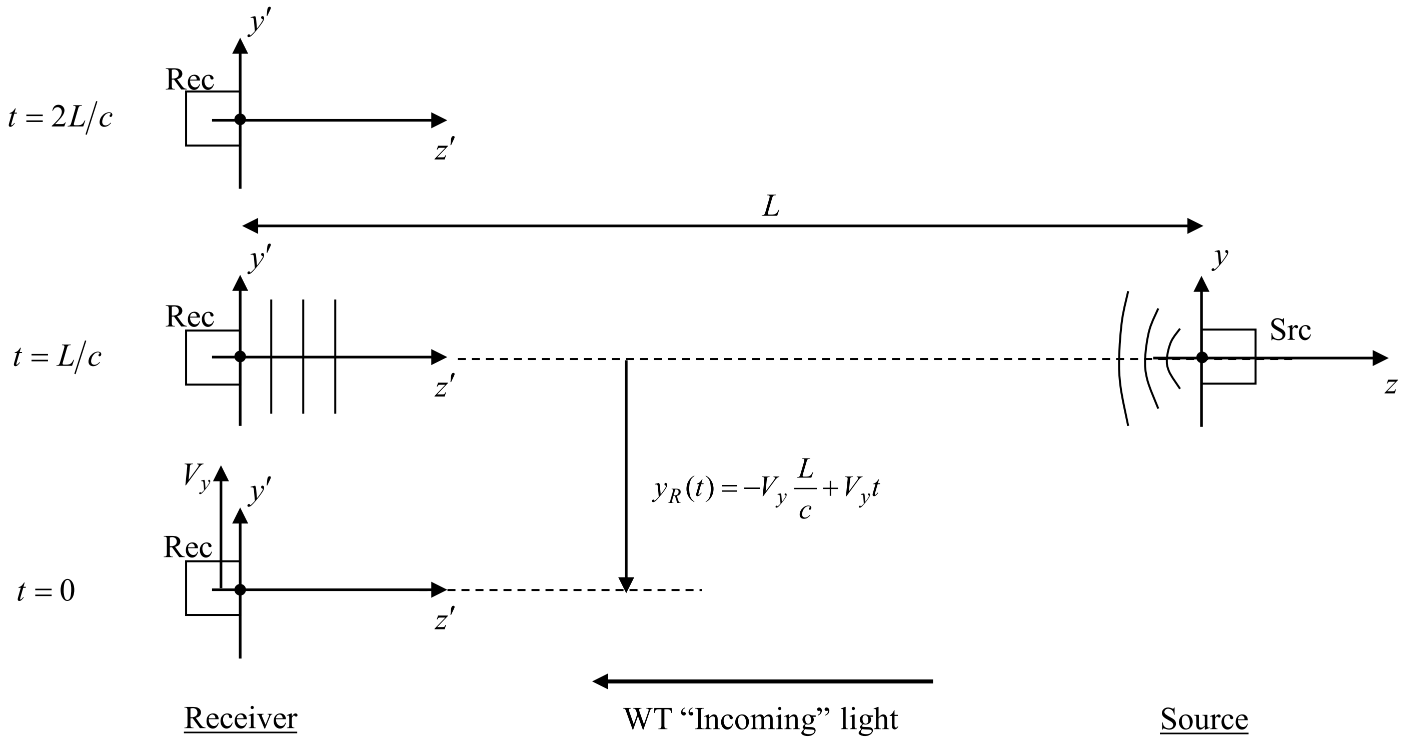
Propagation scenario and Analysis 1

- **In rest frame of source, let:**
  - **Source emit initially collimated beam with 0 tilt, or on-axis spherical wave**
  - **Receiver move transverse to propagation direction, with offsets and timing as shown**



1) **Provisional conclusion:** analysis 1 implies that measured tilt (avg) of wavefront reported by sensor at $t = L/c$ should be $\theta'_y = 0$.
(This was the logic assumed by the users mentioned on the Background slide).

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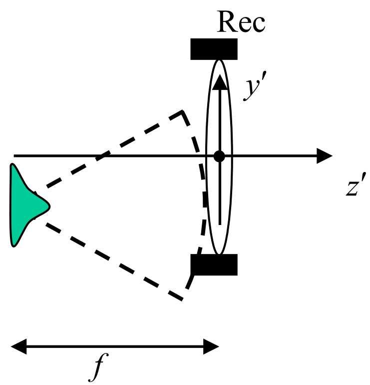
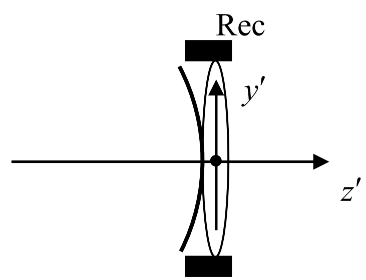
Analysis 2

- **Before inquiring what tilt WT reports, let's do another thought experiment (analysis 2). Continue to visualize in rest frame of source**

$t = L/c + f/c$



$t = L/c$



1) Explicitly consider time (f/c) required for wavefront to propagate from receiver pupil to image plane.
2) Image plane centroid, y'_c, satisfies

$$\frac{y'_c}{f} = -\frac{V_y \cdot (f/c)}{f} = -\frac{V_y}{c}$$
$$= \theta'_y$$

3) The time at which result (2) is valid is slightly > (L/c), but if $f \ll L$, this is for all practical purposes = (L/c). Note that angle is independent of f.
4) Typical (V_y/c) values: (10m/s) / 3E8m/s = 0.033 urad
(1000m/s) / 3E8m/s = 3.3 urad
5) **Provisional conclusion:** "tilt" prediction from analysis 2 contradicts analysis 1 ??
Suggests a non-unique connection between wavefront tilt and reported centroid ??

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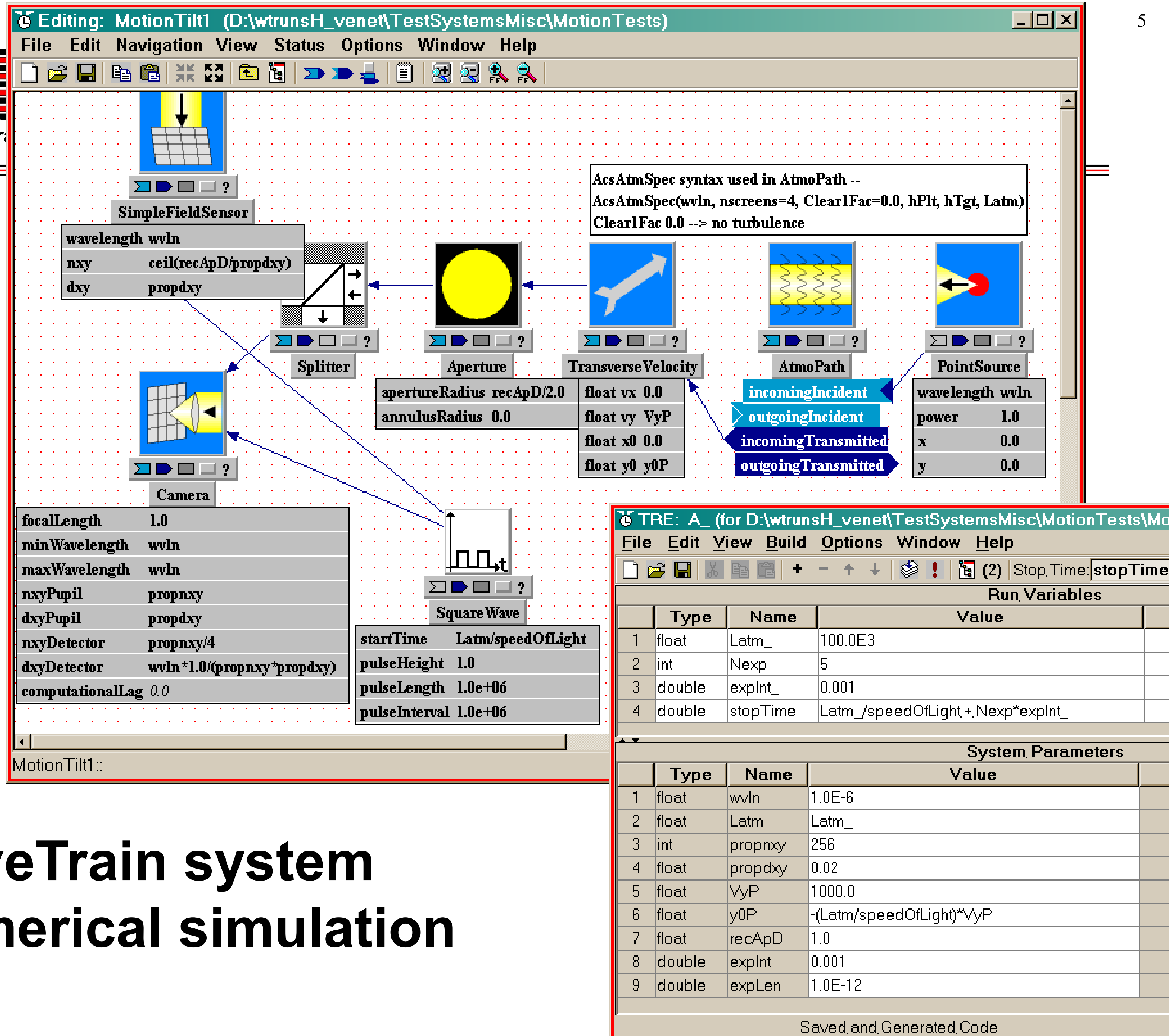

WaveTrain system for numerical simulation

(bv)

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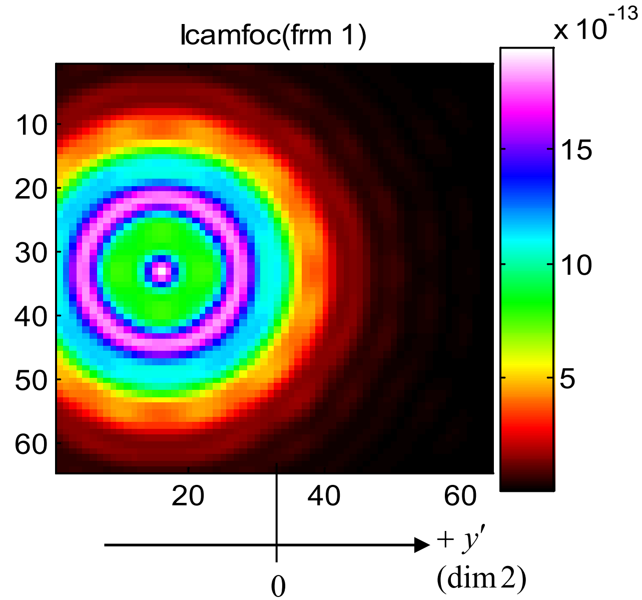
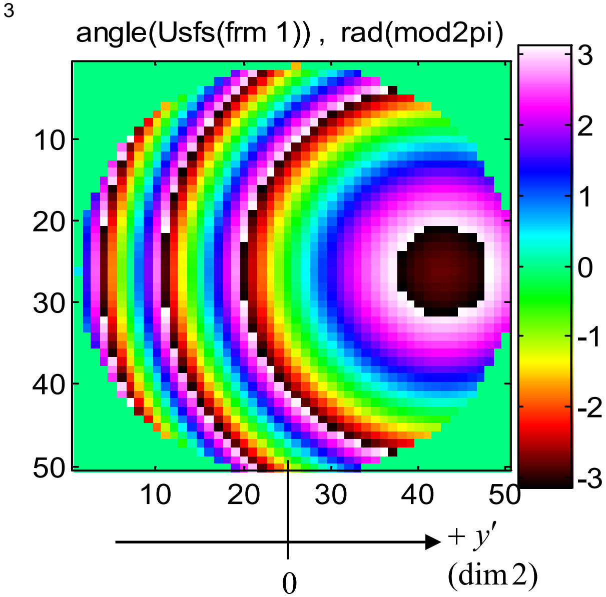
WaveTrain answer for the tilt at $t = L/c$

1) Images from the two WT sensors, at $t = L/c$ (first data frames recorded by the runset timing setup). (Note that WT expLen =1E-12s, negligible)
2) **WT's tilt answer (from both sensors):**
   $\theta'_y = -3.3$ urad
   $= -(V_y/c)$
   WT gives the "analysis 2" answer.

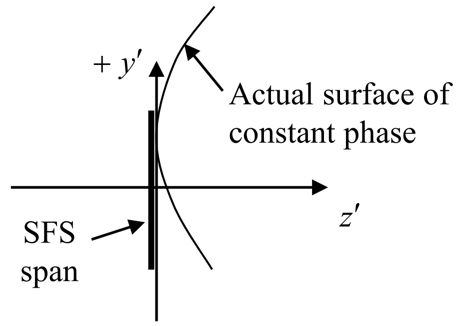
3) NOW, TWO QUESTIONS:
   (A) Is the WT answer right or wrong?
   (B) What logic did WT use? Note that WT got the "analysis 2" answer, but it definitely did not use that logic ...



- Cam foc plane: dxy/foclen = 0.195 urad



- SFS plane: dxy = 0.02 m



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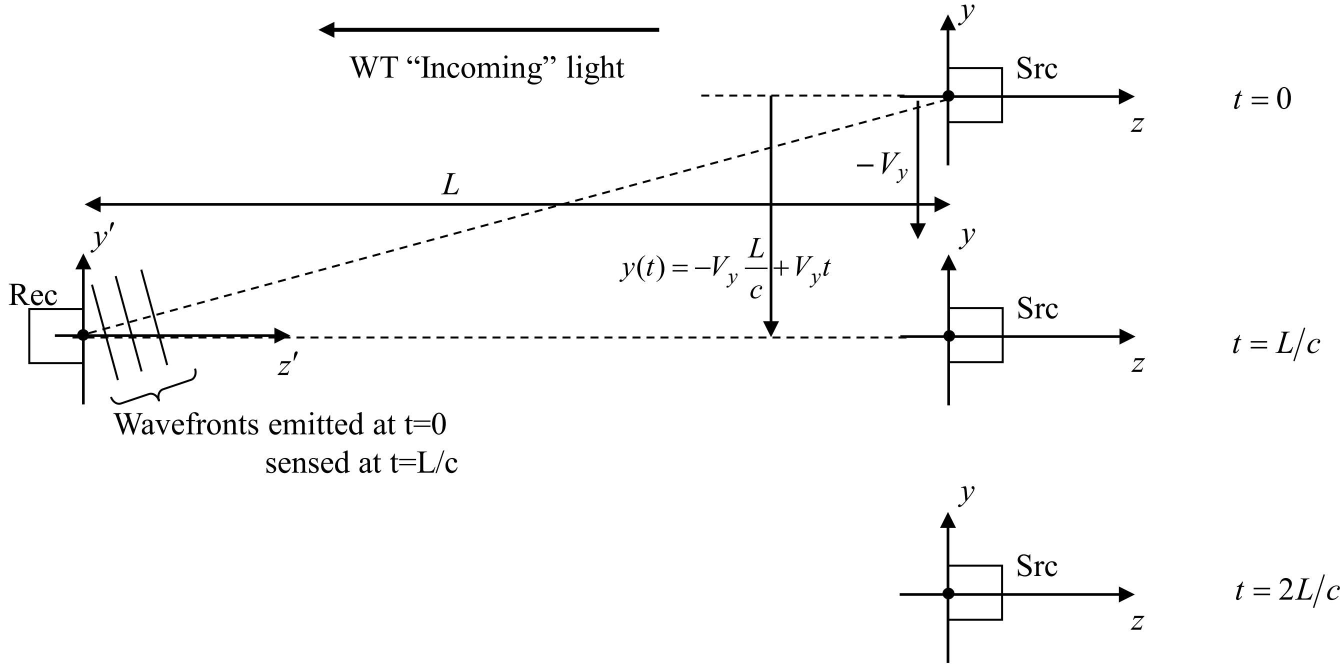
Analysis 3

What WT actually does (and brilliantly, this time...)

- **In rest frame of receiver, we have the picture below**
  - **Analysis concept: at *t=L/c*, receiver senses light emitted by source at earlier (*t*=0) time**
  - **This light presumably has the tilt indicated at the receiver block diagram**



1) **<u>Provisional conclusion</u>:** analysis 3 implies that measured tilt (avg) of wavefront reported by sensor at $t = L/c$ should be $\theta'_y = -(V_y/c)$.
2) Analysis 3 conclusion is same as analysis 2 conclusion, but disagrees with analysis 1.
3) **Concept 3 is essentially the logic built into WaveTrain:** WT's transverse motion manipulations are based on visualizing the problem in the rest frame of the receiver.
4) **Final question:** is analysis 3 correct? After all, analysis 1 looks pretty convincing to some people.

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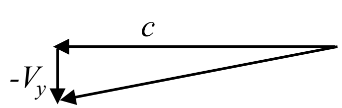
Further analysis and final remarks

- **Analyses 1 and 2: we said those pictures were drawn from the point of view of the source, but really it's a sort of God's eye view of the situation**
- **Formal analytical approach: write a formula that expresses the surface of constant phase in one reference system, and perform t-dependent coordinate transformation to obtain formula for surface in other system**

$$\left.\begin{aligned} &U(x,y,z,t)=\frac{e^{ikr}}{r} \\ &x'=x \\ &y'=y-V_y t \qquad \text{(“Galilean transformations”)} \\ &z'=z \\ &t'=t \end{aligned}\right\} \longrightarrow \quad U'(x',y',z',t')=\cdots$$

- **This approach yields same answer as analyses 2 and 3 (the WT answer)**
- **Another picture frequently used that is equivalent to the formal coord transformation idea is to apply vector velocity addition to a point on the wavefront:**



- **Special Relativity works the problem from the point of view of formal coord transformations, but using the Fitzgerald-Lorentz transformations**
- **Suggested homework:**
  - **WT plane wave case (slew complication)**
  - **Work out the formal Galilean coord coordinate transformation method**