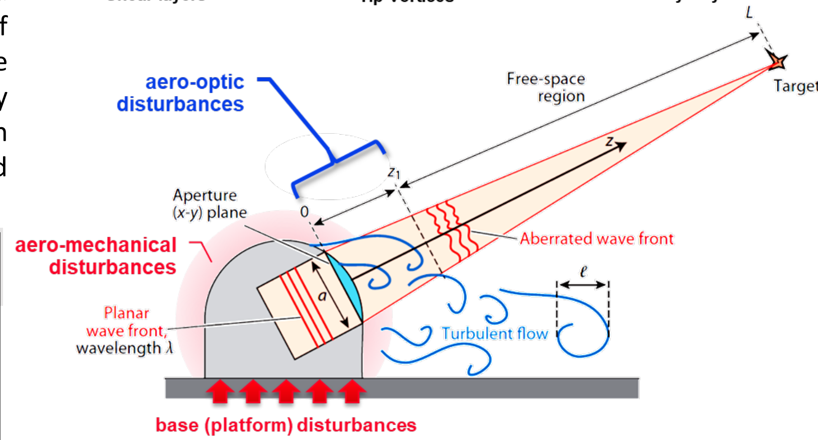
Aero-Optical and Aero-Mechanical Effects

OVERVIEW: Aero-Optical and Aero-Mechanical disturbances can significantly affect laser propagation from airborne platforms. This results in reduced efficiency of laser beam projection for illumination and optical communications. The magnitude of these effects depend on the strength of unsteady turbulent flow field density variations, surface pressure disturbances, the geometry of the physical beam director or surface around the aperture, and the type of flow field encountered. Shear layers, turbulent boundary layers, and wing-tip vortices are generally the most common canonical turbulent flow fields which contribute to aero-optic and aero-mechanical phenomena around airborne beam directors.

Shear-layers Tip-vortices Turbulent Boundary Layers



Fundamentals of Aero-Optics

Gladstone Dale Relation: Index of refraction n of a medium is proportional to density, ρ:

$$n(\vec{x},t) = K_{GD}\rho(\vec{x},t) + 1$$

where K_{GD} is the Gladstone-Dale parameter, which is a weak function of wavelength:

$$K_{GD}(\lambda) = 2.23 \times 10^{-4}\left(1 + 7.52 \times 10^{-15}\lambda^{-1}\right)$$

Optical Path Difference: In compressible turbulent flow around an aircraft, local spatial & temporal variations in density induce differences in the phase along any optical path passing through the region. These are quantified through the integral:

$$\mathrm{OPD}(x,y,t) = K_{GD}\int_0^L \rho(x,y,z,t)dz$$

RMS Wavefront Error It is useful to express variations of OPD as an RMS value. Spatial RMS for an instantaneous wavefront is:

$$\mathrm{OPD}_{\mathrm{RMS}}(t) = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\mathrm{OPD}\left(\mathrm{x}_n,\mathrm{y}_n,t\right)\right)^2}$$

Aero-Optic Disturbance Scaling: The magnitude of an aero-optic disturbance scales proportionally with the product of density, ρ, the square of the Mach number, M, and turret diameter, D_t:

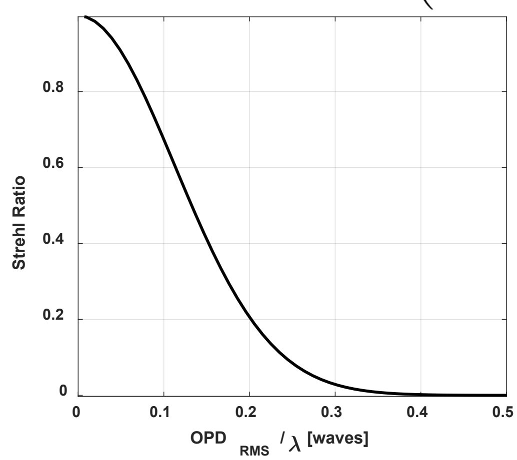
$$\mathrm{OPD}_{\mathrm{RMS}} \propto \left(\rho M^2\right) D_t$$

As a result, $\mathrm{OPD}_{\mathrm{RMS}}$ from one flight condition or beam director may be scaled to other flight conditions and beam director sizes using the relations defined in the table below.

Effect of Aero-Optics in the Far Field

Aero-Optic Strehl Ratio: Using this value, we can estimate the instantaneous effect of the aero-optic phase distortions using the Maréchal approximation:

$$SR_{\text{aero-optic}}(t) = \exp\left(-\left(\frac{2\pi\mathrm{OPD}_{\mathrm{RMS}}(t)}{\lambda}\right)^2\right)$$



$\frac{\mathrm{OPD}_{\mathrm{RMS}}}{\lambda}$	SR
0.05	90.6%
0.10	67.4%
0.15	40.1%
0.20	20.6%
0.25	8.5%
0.30	2.9%
0.40	0.2%
0.50	<0.01%

Parameter	Symbol/Definition
Density	ρ = static density, ρ_{SL} = sea level
Mach	M
Turret Diameter	D_t
Speed of Sound	a
Platform Velocity	$v_p = Ma$

Variables		Scale Factor(s)	Normalized Variable	Scaled to Application
Wavelength	λ	$C_\lambda = \frac{K_{GD}(1\ \mu\mathrm{m})}{K_{GD}(\lambda)}$	***NOTE**: Wavelength scale factor C_λ is used to account for differences in K_{GD} when performing application scaling*	
OPD	ϕ	$C_{AERO} = \left(\frac{\rho}{\rho_{SL}}M^2D_t\right)^{-1}$	$\Phi = \phi\left(\frac{\rho}{\rho_{SL}}M^2D_t\right)^{-1}$	$\phi' = \left(\frac{K_{GD}(\lambda')}{K_{GD}(\lambda)}\right)\left(\frac{\rho'}{\rho}\right)\left(\frac{M'}{M}\right)^2\left(\frac{D_t'}{D_t}\right)\phi$
Aperture Coordinates	$\vec{x} = (x,y)$	$\frac{1}{D_t}$	$\vec{X} = (X,Y) = \frac{\vec{x}}{D_t}$	$\vec{x}' = \frac{D_t'}{D_t}\vec{x}$
Frequency	f	$\frac{D_t}{v_p}$	$F = \left(\frac{D_t}{v_p}\right)f$	$f' = \left(\frac{v_p'}{v_p}\right)\left(\frac{D_t}{D_t'}\right)f$
Time	t	$\frac{v_p}{D_t}$	$\tau = \left(\frac{v_p}{D_t}\right)t$	$t' = \left(\frac{v_p}{v_p'}\right)\left(\frac{D_t'}{D_t}\right)t$

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Aero-Optical and Aero-Mechanical Effects

Fundamental Equations: Aero-Mechanical Effects

Equation of Motion: The vibration of an object is governed by a variation of the standard equation of motion that accounts for deformation of the object. Vibration can be excited by pressure loading, base disturbances or local forcing.

$$m\ddot{x} + c\dot{x} + kx = pA + MR\ddot{x_B} + F$$

Modal Form of Motion: Vibration is often decomposed into a modal form where each mode has a characteristic frequency.

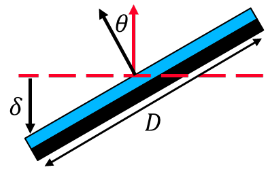
$$\ddot{u} + 2\zeta\omega\dot{u} + \omega^2 u = \phi^T pA + p_M\ddot{x_B} + \phi^T F$$

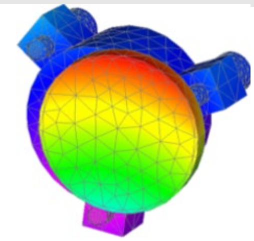
Conversion to Jitter: Vibration represents as mechanical jitter when its modes move optics in the system and can be computed using the small angle approximation.

$$\theta(f) = \frac{\delta}{D}(f) = \frac{u}{F}(f)\frac{\phi}{D}F$$

Flight Condition Scaling: Aero-mechanical jitter scales with pressure on the beam director surface, which is characterized by the dynamic pressure, *q*. It does not scale by turret diameter, as structure stiffness and applied force scale the same.

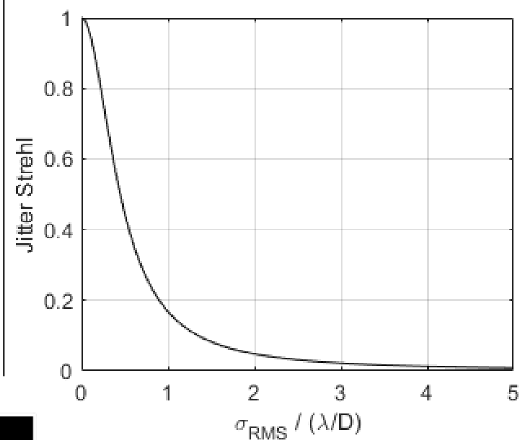
$$\theta_{RMS} \propto q \qquad\qquad q = \frac{1}{2}\rho {v_p}^2$$





Jitter Strehl Ratio: Jitter Strehl ratio can be obtained by assuming it spreads a Gaussian beam into a larger, but still Gaussian shape:

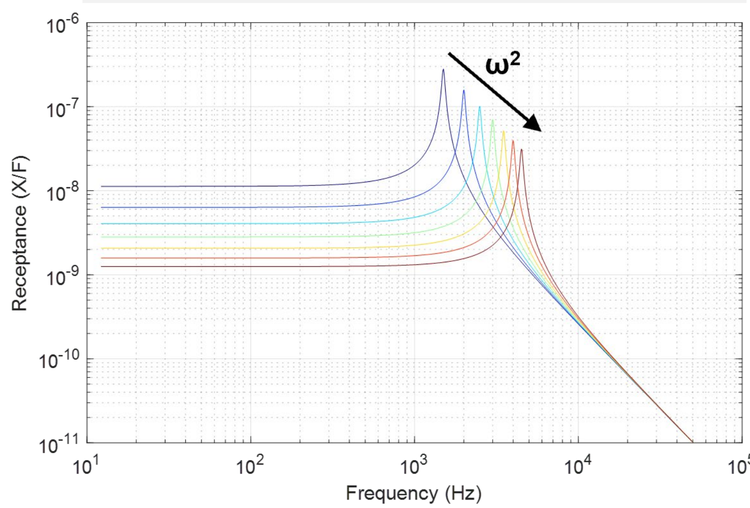
$$SR_{\text{aero-jitter}} = \left(1 + \frac{\pi^2}{2}\frac{\sigma_{\text{aero}}^2 + \sigma_{\text{mech}}^2}{(\lambda/D)^2}\right)^{-1}$$



$\frac{\sigma}{\lambda/D}$	SR
0.10	95.3%
0.25	76.4%
0.50	44.7%
0.75	26.5%
1.00	16.9%
1.50	8.3%
2.00	4.8%
4.00	1.25%

Beam Director Jitter Scaling Parameter	Equation	Proportion to Length Scale	Beam Director Scale Percentage		
			25%	50%	100%
Length	$L = L$	L	0.25	0.50	1.00
Area	$A = L^2$	L^2	0.06	0.25	1.00
Volume	$V = L^3$	L^3	0.016	0.13	1.00
Mass	$m = V$	L^3	0.016	0.13	1.00
Vibe Mode Frequency	$f_M = L^{-1}$	L^{-1}	4.00	2.00	1.00
Vibe Mode Amplitude	$C_\varphi = m^{-1/2}$	$L^{-3/2}$	8.00	2.83	1.00
Vibe Mode TF	$C_f = f^{-2}$	L^2	0.06	0.25	1.00
Aero Frequency	$f_F = L^{-1}$	L^{-1}	4.00	2.00	1.00
Base Motion Jitter Ratio	$J_B = \frac{C_f C_\varphi \sqrt{m}}{L}$	L	0.25	0.50	1.00
Pressure Jitter Ratio	$J_F = \frac{C_f {C_\varphi}^2 A}{L}$	*no dependence*	1.00	1.00	1.00

Vibration Mode Transfer Function (δ/F)



Nomenclature

n	index of refraction	K_{GD}	Gladstone-Dale constant	D	aperture diameter	p	pressure
ρ	density	OPD	optical path difference	λ	wavelength	ω	mode frequency
M	Mach number	x, y	aperture coordinates	f	frequency	ζ	mode damping
D_t	turret diameter	z	distance along path	m	mass	F	Force

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