Optical Propagation Through The Atmosphere

OVERVIEW: Atmospheric turbulence fluctuations significantly affect high-resolution imaging sensors and can reduce efficiency of laser beam projection for illumination and optical communications. The magnitude of these effects depends on the strength of turbulence and the distance from aperture to target, as well as the wavelength and aperture diameter of the optical system.

Nomenclature

n	index of refraction	C_n^2	strength of turbulence	D	aperture diameter
κ	spatial frequency	λ	wavelength	x	position in aperture
Φ	power spectrum	L	slant range	V	transverse speed along path
l_0	inner scale	z	distance along path		
L_0	outer scale				

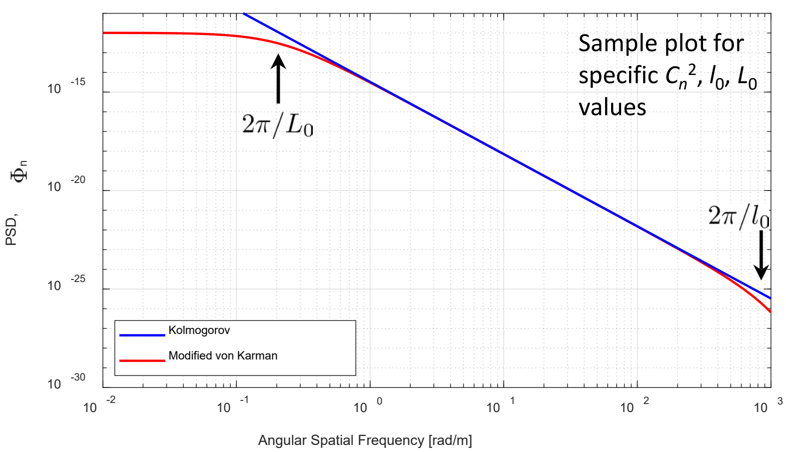
Fundamental Equations: Turbulence

Kolmogorov Model: Turbulence-induced index fluctuations follow a simple power spectrum model:

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3}$$

Inner and Outer Scale: Kolmogorov model only applies at intermediate values of spatial frequency. A common-used formula incorporates inner scale and outer scale as cutoffs:

$$\Phi_n(\kappa) = 0.033 C_n^2 \frac{1}{\left[\kappa^2 + \left(2\pi/L_0\right)^2\right]^{11/6}} \exp\left[-\left(\frac{\kappa}{2\pi/l_0}\right)^2\right]$$



Atmospheric Coherence Diameter, r_0: Also called "Fried's parameter", gives the limit to resolving or focusing ability through the turbulence volume. When $D/r_0 > 1$, resolution is determined by r_0 instead of aperture, D. The relative aperture, D/r_0 governs many phase-related propagation effects, including turbulence-induced jitter and wavefront error.

spherical wave r_0:

$$r_0 = \left[\frac{2.91}{6.88}\left(\frac{2\pi}{\lambda}\right)^2 \int_0^L C_n^2(z)(1 - z/L)^{5/3} dz\right]^{-3/5}$$

uniform C_n^2 over path:

$$r_0 = \left[\frac{2.91}{6.88} \cdot \frac{3}{8}\left(\frac{2\pi}{\lambda}\right)^2 L\, C_n^2\right]^{-3/5}$$

= 0.159

Isoplanatic angle, Θ_0: Angle over which turbulence-induced phase effects are correlated. For angular offsets greater than Θ_0, *anisoplanatic* turbulence effects are observed in compensation.

spherical wave isoplanatic angle:

$$\theta_0 = \left[2.91\left(\frac{2\pi}{\lambda}\right)^2 L^{5/3} \int_0^L C_n^2(z)\left(z/L\right)^{5/3} dz\right]^{-3/5}$$

uniform C_n^2 over path:

$$\theta_0 = \left[2.91 \cdot \frac{3}{8}\left(\frac{2\pi}{\lambda}\right)^2 L^{8/3} C_n^2\right]^{-3/5} = 0.3143\left(\frac{r_0}{L}\right)$$

= 1.093

Rytov number: Log-amplitude variance for propagation through turbulence, an index to the magnitude of scintillation or beam break-up due to propagation.

spherical wave Rytov number:

$$\sigma_\chi^2 = 0.563\left(\frac{2\pi}{\lambda}\right)^{7/6} L^{5/6} \int_0^L C_n^2(z)\left(z/L\right)^{5/6}\left(1 - z/L\right)^{5/6} dz$$

uniform C_n^2 over path:

$$\sigma_\chi^2 = 0.563 \cdot 0.2205\left(\frac{2\pi}{\lambda}\right)^{7/6} L^{11/6} C_n^2$$

= 0.124

Greenwood frequency, f_G: An integrated measure of the rate of temporal disturbance fluctuation. Refers to an analytic estimate of the required correction bandwidth needed by adaptive-optics components to compensate respectably for the turbulence

$$f_G = \left[0.102\left(\frac{2\pi}{\lambda}\right)^2 \int_0^L C_n^2[h(z)]|V(z)|^{5/3} dz\right]^{3/5}$$

Tyler frequency, f_T: Similar to Greenwood frequency, but relates to the tilt component of turbulence-induced wavefronts

$$f_T = 0.368 D^{-1/6} \lambda^{-1}\left[\int_0^L C_n^2[h(z)]|V(z)|^2 dz\right]^{1/2}$$

$$\sigma_\phi^2 = \left(f_G/f_{BW}\right)^{5/3}, \qquad \sigma_j^2 = \left(f_T/f_{BW}\right)^2\left(\lambda/D\right)^2$$

λ scaling: $\dfrac{r_0'}{r_0} = \dfrac{\theta_0'}{\theta_0} = \left(\dfrac{\lambda'}{\lambda}\right)^{6/5}, \quad \dfrac{\sigma_\chi^{2\,\prime}}{\sigma_\chi^2} = \left(\dfrac{\lambda'}{\lambda}\right)^{-7/6}$

4900 Lang Ave. NE, Suite 100
Albuquerque, NM 87109
505-245-9970

1360 Technology Ct, Suite 200
Dayton, OH 45430
937-684-4100

1312 Underwood Rd.
La Porte, TX 77571
281-476-4716

delta@mza.com
wavetrain@mza.com
share@mza.com

Optical Propagation Through The Atmosphere

OVERVIEW: Light scatters from and is absorbed by air molecules (N_2, O_2, Ar, CO_2, etc.) and aerosols. This reduces the amount of light received by an optical sensor or laser power delivered to a target. The magnitude of these effects depends on the atmospheric conditions and the distance from the aperture to the target, as well as the wavelength.

Nomenclature

σ_{mol}	molecular scatter coefficient	μ	total extinction coefficient
σ_{aer}	aerosol scatter coefficient	T	transmission
k_{mol}	molecular absorption coefficient	$I(z)$	optical irradiance
k_{aer}	aerosol absorption coefficient		

Fundamental Equations: Scattering and Absorption

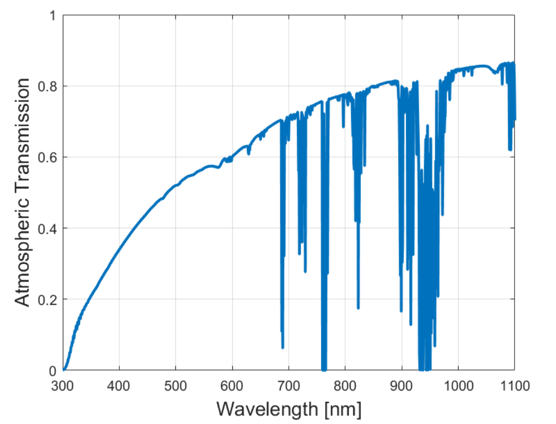
Total Extinction: The total extinction coefficient is the sum of the molecular and aerosol scattering and absorption:

$$\mu = \underbrace{\sigma_{mol} + \sigma_{aer}}_{\textbf{scattering}} + \underbrace{k_{mol} + k_{aer}}_{\textbf{absorption}}$$

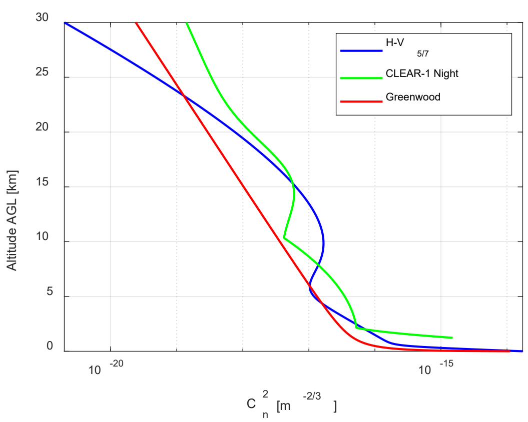
Light Attenuation: The total extinction coefficient is the sum of the molecular and aerosol scattering and absorption:

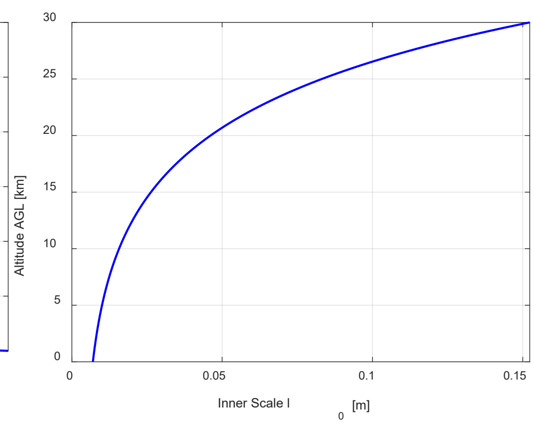
$$T = \exp\left[-\int_0^L \mu(z,\lambda)\,dz\right],\ I(L) = T\,I(0)$$

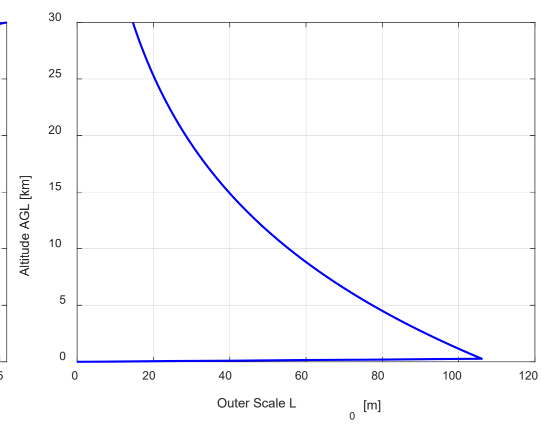
uniform C_n^2 over path: $$T = \exp\left[-\int_0^L \mu(z,\lambda)\,dz\right],\ I(L) = T\,I(0)$$

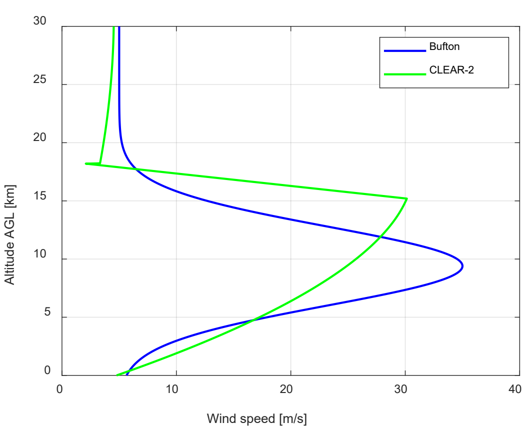


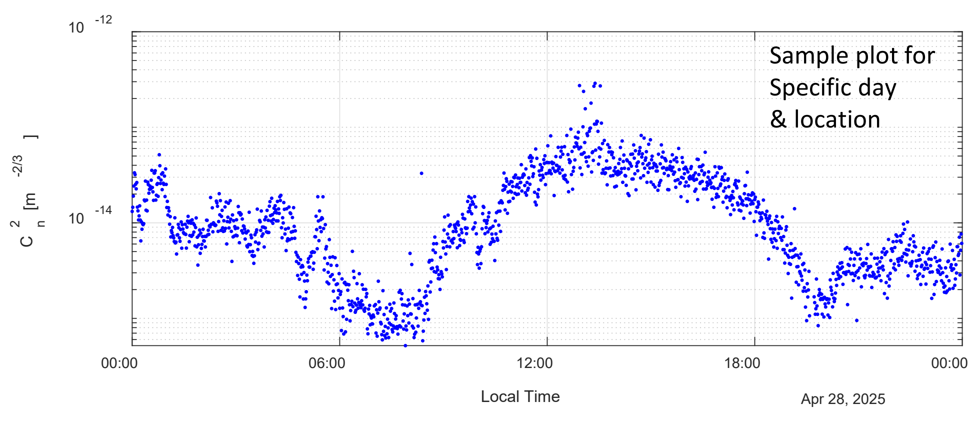
Popular Models and Helpful Plots











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