Laser System Performance Modeling & Simulation

OVERVIEW: Scaling laws are analysis methods for estimation of target irradiance properties through consideration of laser propagation metrics. Scaling laws support a diverse set of parameter assumptions and include a variety of system, propagation, and threat-induced effects based on theoretical analysis or empirical assumptions.

Nomenclature

D	sperture diameter	σ_χ^2	Rytov number	S	Strehl ratio
λ	wavelength	f_G	Greenwood frequency	S_h	higher-order Strehl
r_0	Fried parameter	f_T	Tyler frequency	σ_j	jitter
θ_0	isoplanatic angle	$f_{\rm bw}$	control bandwidth	σ_h	higher-order beam spread

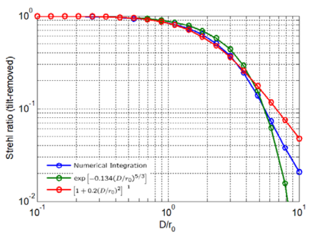
Fundamental Effects

Phase Variance: Increasing turbulence strength results in increasing variance in the atmospheric phase that induces both tilt and higher-order effects.
Characterized by the Fried parameter.
Open-loop jitter:

$$\sigma_j^2 = 0.1816(D/r_0)^{5/3}(\lambda/D)^2$$

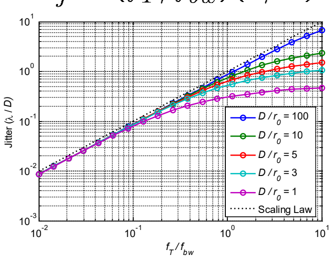
Higher-order (tilt-removed) Strehl:

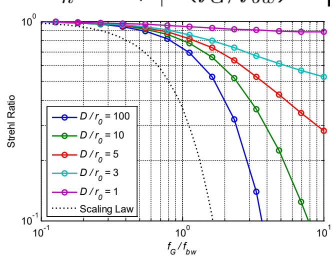
$$S_h = \exp\left[-0.134(D/r_0)^{5/3}\right]$$



Temporal Effects: Limitations in correcting for high-frequency turbulence due to finite temporal response of the control system. Affects both tilt and higher order compensation.

$$\sigma_j = (f_T/f_{bw})(\lambda/D) \qquad S_h = \exp\left[-(f_G/f_{bw})^{5/3}\right]$$





Scintillation: Amplitude fluctuations characterized by the log-amplitude variance (Rytov number).

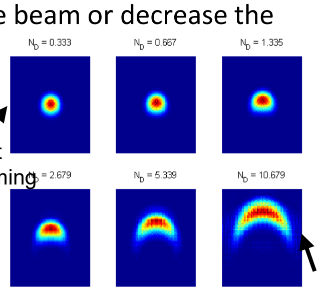
$$S = \exp(-\sigma_\chi^2)$$

Induces track scintillation error and "hidden" phase effects in WFS reconstructors

Thermal Blooming: Reduction in peak irradiance due to heating of the air by high energy lasers which induces a defocus. Mitigated by wind moving the heated air out of the propagation path and by any effects that spread the beam or decrease the power density in the beam.
Higher-order effect characterized by t distortion number:

$$N_d = \frac{4\sqrt{2}k_0 P}{\rho_0\, C_p}\int_0^L dz\, \frac{n_T(z)\alpha(z)}{V(z)D(z)} \times \exp\left\{-\int_0^z dz'\, [\alpha(z') + s(z')]\right\}$$

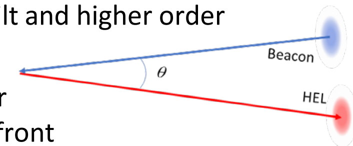
$$S = [1 + f(N_d)]^{-1}$$



Least blooming

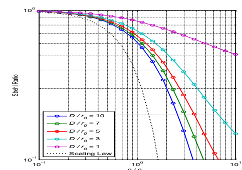
Most blooming

Angular Anisoplanatism: Limitations in correcting turbulence when the measurement uses a beacon in a different direction than the HEL propagation direction. Characterized by the isoplanatic angle. Affects both tilt and higher order compensation.



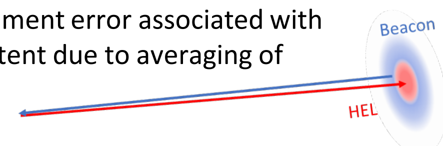
Higher-order anisoplanatism is error induced when corrections for wavefront error are applied to a different propagation direction.

$$S = \exp\left[-(\theta/\theta_0)^{5/3}\right], \text{ for } (D/r_0)/(\theta/\theta_0) \to \infty$$

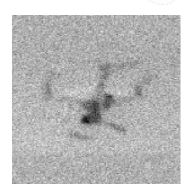


Tilt anisoplanatism is a tracking error associated with measuring tilt in one direction and applying the correction in a different direction.

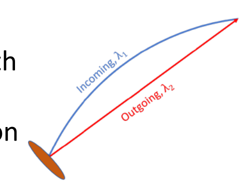
Extended Beacon: Measurement error associated with a beacon of finite spatial extent due to averaging of turbulence contributions along the path



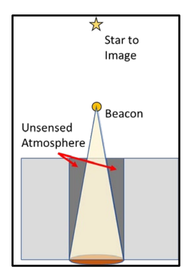
Sensor noise: Measurement error associated with finite signal emitted from source or reflected from target relative to sensor noise. Can affects both tilt and higher-order compensation.



Chromatic Dispersion: Compensation error associated with measuring at one wavelength to compensate another wavelength—sampling of different paths along propagation volume due to refraction.



Focal Anisoplanatism: Compensation error associated with measuring turbulence from a source that does not sample the entire propagation volume. Important for laser guide stars.



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MZA Laser System Performance Modeling & Simulation

Fundamental Equations: Vacuum Path Performance

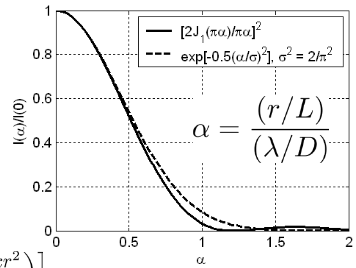
Free Space Propagation: Modeling the diffraction limited beam profile from a uniform beam (Airy pattern).

$$I(r) = \varepsilon P \frac{\pi D^2}{4\lambda^2 L^2}\left[\frac{2J_1(\pi\frac{rD}{\lambda L})}{\pi\frac{rD}{\lambda L}}\right]^2$$

$$\begin{aligned} \varepsilon &= \text{attenuation} \\ P &= \text{laser power (W)} \\ D &= \text{transmitter diameter (m)} \\ \lambda &= \text{wavelength (m)} \\ L &= \text{slant range (m)} \end{aligned}$$

Gaussian fit to Airy Disk: Convenient to model as a Gaussian.

$$\hat{I}(\alpha) = \frac{1}{2\pi\sigma_R^2}\exp\left[-\frac{\alpha^2}{2\sigma_R^2}\right]$$



$$\alpha = \frac{(r/L)}{(\lambda/D)}$$

Gaussian Beam Propagation: Infinite Gaussian beam propagation

$$U(r,z) = \frac{1}{\Theta_0 + \jmath\Lambda_0}\exp\left(-\frac{r^2}{W^2}\right)\exp\left[\jmath\left(kz - \frac{kr^2}{2F}\right)\right]$$

$$\begin{aligned} \Theta_0 &= 1 - \frac{z}{F_0} \\ \Lambda_0 &= \frac{2z}{kW_0^2} \\ W &= W_0\sqrt{\Theta_0^2 + \Lambda_0^2} \end{aligned}$$

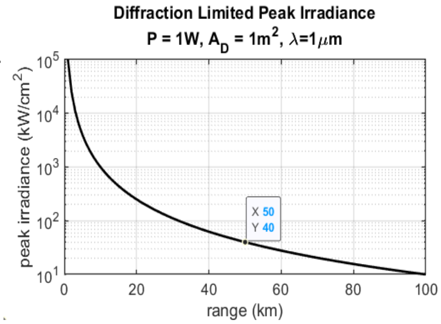
$$I(r,z) = \frac{1}{\Theta_0^2 + \Lambda_0^2}\exp\left(-\frac{2r^2}{W^2}\right)$$

Uniform beam diameter that gives same spot size as a focused Gaussian with waist W_0

$$W = \frac{\lambda L}{\pi W_0} = 2\sigma_U L = \frac{2\sqrt{2}}{\pi}\frac{\lambda L}{D}$$

$$D = 2\sqrt{2}W_0$$

Peak Irradiance: DL peak irradiance scales with laser power, aperture area, and inversely to wavelength squared



$$I_{\mathrm{pk}} = \frac{P}{2\pi\sigma^2} = \frac{A_D P}{\lambda^2 L^2}$$

Power in Bucket: Power in Bucket or PIB is a convenient measure of laser weapon performance. Used frequently in lethality tables. PIB for a bucket with diameter *d:*

$$\mathrm{PIB} = 2\pi\int_0^{d/2} rI(r)\mathrm{d}r = P\left[1 - \exp\left(-\frac{A_d A_D}{\lambda^2 L^2}\right)\right]$$

Beam Quality: Measure of how tightly a beam can be focused – also referred to as M2. BQ > 1 means the beam diverges beyond normal diffraction.

½ angle Gaussian divergence

$$\sigma = \frac{\sigma_{\mathrm{diff}}}{\mathrm{BQ}} \qquad S_{h\mathrm{BQ}} = \frac{1}{\mathrm{BQ}^2} \qquad \theta_{\mathrm{div}} = M^2\frac{\lambda}{\pi W_0}$$

Atmospheric Effects: Popular Models

Open Loop Propagation: Jitter and Strehl (long term and higher-order) assuming no tracking or adaptive optics compensation.

$$\sigma_{jOL} = \sqrt{0.449\frac{4}{\pi^2}\left(\frac{D}{r_0}\right)^{5/3}\left(\frac{\lambda}{D}\right)^2} \qquad \mathcal{H}(\Omega) = \exp\left\{-3.44\left(\frac{\lambda\Omega}{r_0}\right)^{5/3}\left[1 - \alpha\left(\frac{\Omega}{\Omega_0}\right)^{1/3}\right]\right\}$$

$$S = \frac{\int d\Omega\mathcal{H}(\Omega)\mathcal{P}(\Omega)}{\int d\Omega\mathcal{P}(\Omega)}$$

long/short (0/1) exposure

pupil autocorrelation function

cutoff frequency

Higher Order Strehl: Higher order (or tilt-removed) Strehl defined as the ratio of areas.

$$S_h = \frac{A_d}{A} = (1 + \Delta A/A_d)^{-1} \qquad \Delta A \equiv A - A_d$$

Maréchal Approximation: Higher order Strehl as a function of phase variance.

$$S_{h_k} \simeq \exp\left(-\sigma_{\phi_k}^2\right)$$

Area Addition: Multiple (*N*) higher order Strehl effects combined via area addition.

$$S_h = \left(1 - N + \sum_{k=1}^{N} S_{h_k}^{-1}\right)^{-1}$$

Jitter: Standard deviation of the beam motion at target. Multiple jitter effects combined via RSS. Jitter equivalent Strehl – increase in beam area due to jitter

$$\sigma_j = \sqrt{\sum_1^N \sigma_i^2} \qquad S_j = 1 + \frac{\pi^2}{2}\sigma_j^2$$

Spot Size: Total long-term spot size at target is the "rollup" of all jitters and higher order Strehls.

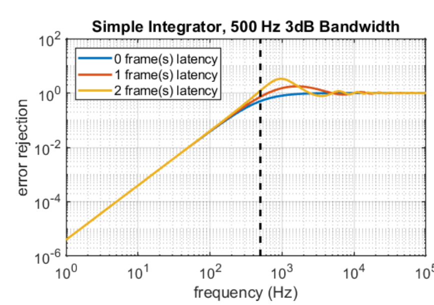
Higher order (tilt-removed) spot

$$\sigma_T = \sqrt{\sigma_j^2 + \sigma_h^2} \qquad \sigma_h = \frac{\sigma_{\mathrm{diff}}}{\sqrt{S_h}}$$

Peak Irradiance: Peak irradiance is the same equation as above with the beam spread replaced by the long-term beam spread rollup and reduced by the all losses, τ, including atmospheric transmission.

$$I_{\mathrm{pk}} = \frac{P\tau}{2\pi\sigma_T^2} \qquad \tau = \prod_1^N \tau_i$$

Closed-loop Control: Residual jitter and Strehl computed as the integral of the error rejection function times the disturbance PSD.



Simple Integrator Control:

$$\mathrm{ERJ}(f) = \left[1 + (f_{\mathrm{BW}}/f)^2 - 2(f_{\mathrm{BW}}/f)\sin(2\pi f\Delta t)\right]^{-1}$$

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